

# Study Problems: Final

1. Ethylene 'mer' :  $C_2H_4$  ; mol. wt.  $28.054 \text{ g/mole}$   
 The polyethylene has polymer molecules having five different molecular weights.

Type	Number	$X_N$ , fraction	mol. wt.	$X_W$ , fraction	$X_W$ (mol. wt)
1	E*	0.200	2805.4	0.0667	187
2	E	0.200	5610.8	0.1333	748
3	E	0.200	8416.2	0.2000	1683
4	E	0.200	11221.6	0.2667	2993
5	E	0.200	14027	0.3333	4676
total: 5E		1.000	—	1.000	10,287

\* Equal number.

number average mol. wt. =  $M_N$

$$M_N = 0.2(2805.4 + 5610.8 + \dots + 14027) = 8416$$

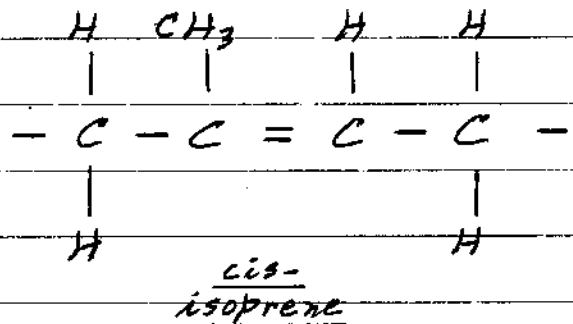
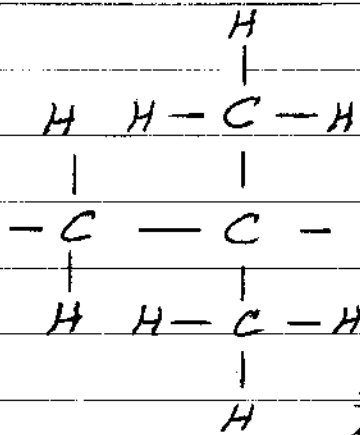
mass-average mol. wt. =  $M_W$

$$M_W = 0.06667(2805.4) + \dots + 0.3333(14027)$$

or  $M_W = 10,287$  ;  $DP = \frac{10287}{28.054} = 367$

A smaller DP is  $M_N$ /mer mol. wt. ; it is 300 only.

2.



$$\text{mol. wt. isobutylene mer } (-C_4H_8-) = 56.108 \frac{\text{g}}{\text{mole}}$$

$$\text{mol. wt. isoprene } (-C_4H_6CH_2-) \text{ mer} = 68.119 \text{ g/mole}$$

Co-polymer (random mix):

$$\text{mole-fraction of isoprene} = 0.75$$

$$\begin{aligned} \text{mol. wt. of co-polymer 'mer'} &= 0.75(68.119) + 0.25(56.108) \\ &= 65.12 \text{ g/mole} \end{aligned}$$

$$\text{'number-average' mol. wt.} = 1500 \times DP$$

$$= \underline{97,675 \text{ g/mole}}$$

3. relaxation time ( $\tau$ ) = 45 days

modulus of elasticity ( $E$ ) = 70 MPa

At  $100^\circ\text{C}$ :

compressive strain,  $\epsilon_0 = 0.05$

Initial condition:

compressive stress,  $\sigma_0 = E\epsilon_0 = \underline{3.5 \text{ MPa}}$

Due to viscoelastic relaxation, at any time  $t$ , with temperature at  $100^\circ\text{C}$ ,

$$\frac{d\sigma}{\sigma} = -\frac{dt}{\tau}$$

Integrating with  $\sigma = \sigma_0$  at  $t = 0$ ,

$$\sigma(t) = \sigma_0 \cdot \exp(-t/\tau)$$

(a) At  $t = 1$  day:

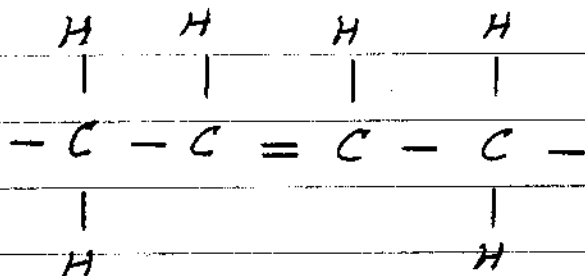
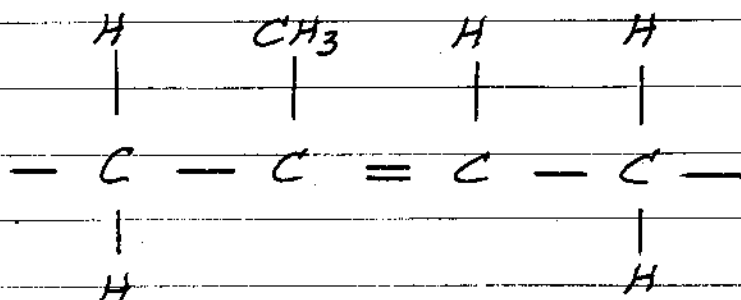
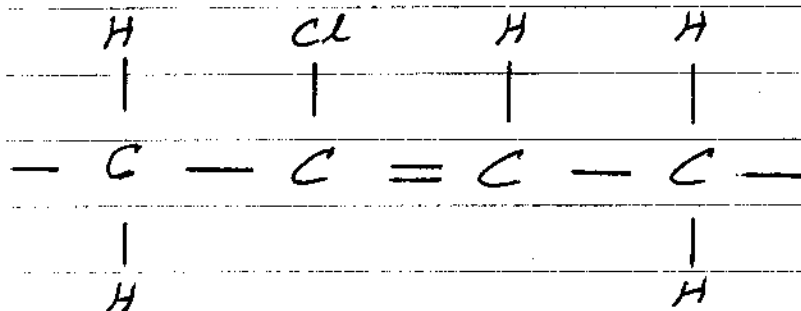
$$\sigma = 3.5 \exp(-1/45) = \underline{3.423 \text{ MPa}}$$

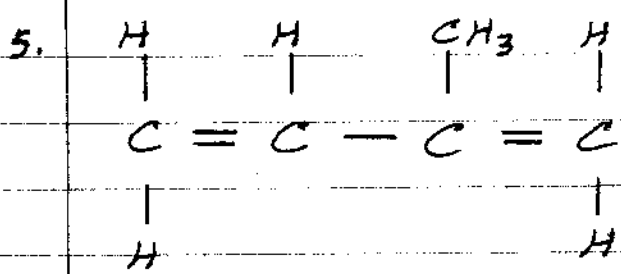
(b) At  $t = 30$  days:

$$\sigma = 3.5 \exp(-2/3) = \underline{1.797 \text{ MPa}}$$

(c)  $\sigma = 3.5 \exp(-365/45) = \underline{1050 \text{ Pa}}$  : 365 days

4.

Butadiene:mol. wt. 54.092cis-Isoprene:mol. wt. 68.119Chloroprene:mol. wt. 88.537



isoprene monomer  
 mol. wt.  $68.119 \frac{\text{g}}{\text{mole}}$   
 $\approx 68 \text{ g/mole}$

Consider 400g of vulcanized polyisoprene:

mass of isoprene =  $400 (0.68) = 272 \text{ g}$

isoprene 'mer' =  $\frac{272}{68} = 4$

mass of sulfur =  $400 (0.32) = 128 \text{ g}$

mol. wt. of S =  $32 \text{ g/mole}$

Sulfur atoms (to cross-link) =  $\frac{128}{32} = 4$

There are 4 sulfur cross-links per 4 mers;  
 hence, one S per mer.

6. polychloroprene  $(-C_4H_5Cl-)$  = 10 kg

mol. wt. of a 'mer' =  $88.537 \text{ g/mole}$

number of chloroprene mers =  $\frac{10^4}{88.537} = 113$

amount of S =  $0.72 \text{ kg}$

number of S atoms =  $\frac{0.72 \times 10^3}{32} = 22.5$

For a fully-cross-linked polymer, there would  
 be 113 sulfur atoms (1 per mer); at 22.5 S-  
 atoms per 113 chloroprene mers,

fraction cross-linked =  $\frac{22.5}{113} = 0.20$

7. In general, for the  $i$ -th material,

area of cross-section =  $A_i = \frac{F}{\sigma_i} = \frac{\text{Load}}{\sigma_i}$

where  $\sigma_i$  = yield strength of the  $i$ th alloy.

At a specified length  $L = 0.25 \text{ m}$ , the respective weights can be found using

$$w_i = (\text{area}) \text{ length } (\text{density}) = 0.25 A_i \rho_i$$

where  $\rho_i$  is the density of the  $i$ -th alloy.

$$\text{Load, } F = 44,400 \text{ N}$$

Thus

$$w_{Al} = 0.25 F \frac{2.7}{275} = 2.455 \times 10^{-3} F \equiv 1.07$$

$$w_{Br} = 0.25 F \frac{8.5}{415} = 5.121 \times 10^{-3} F \equiv 2.23$$

$$w_{St} = 0.25 F \frac{7.9}{860} = 2.300 \times 10^{-3} F \equiv 1.0$$

Thus, steel offers the lowest weight with aluminum a close second. One can show the respective data are as follows:

$A_i$ $\text{m}^2$	diameter $\text{m}$	wt., $w_i$ in kg	Alloy $i$
0.000161	0.01432	0.109	Al
0.000107	0.01167	0.2275	Brass
0.00005163	0.008108	0.102	Steel

8. Fig. 10.14 (W.D. Callister, Jr.):

Austenite  $\rightarrow$  pearlite transformation:

- i) begins at  $t \approx 2.4 \text{ sec}$
- ii) 50% completed at  $t \approx 5.4 \text{ sec}$
- iii) 100% completed at  $t \approx 12.8 \text{ sec}$

Problem 8

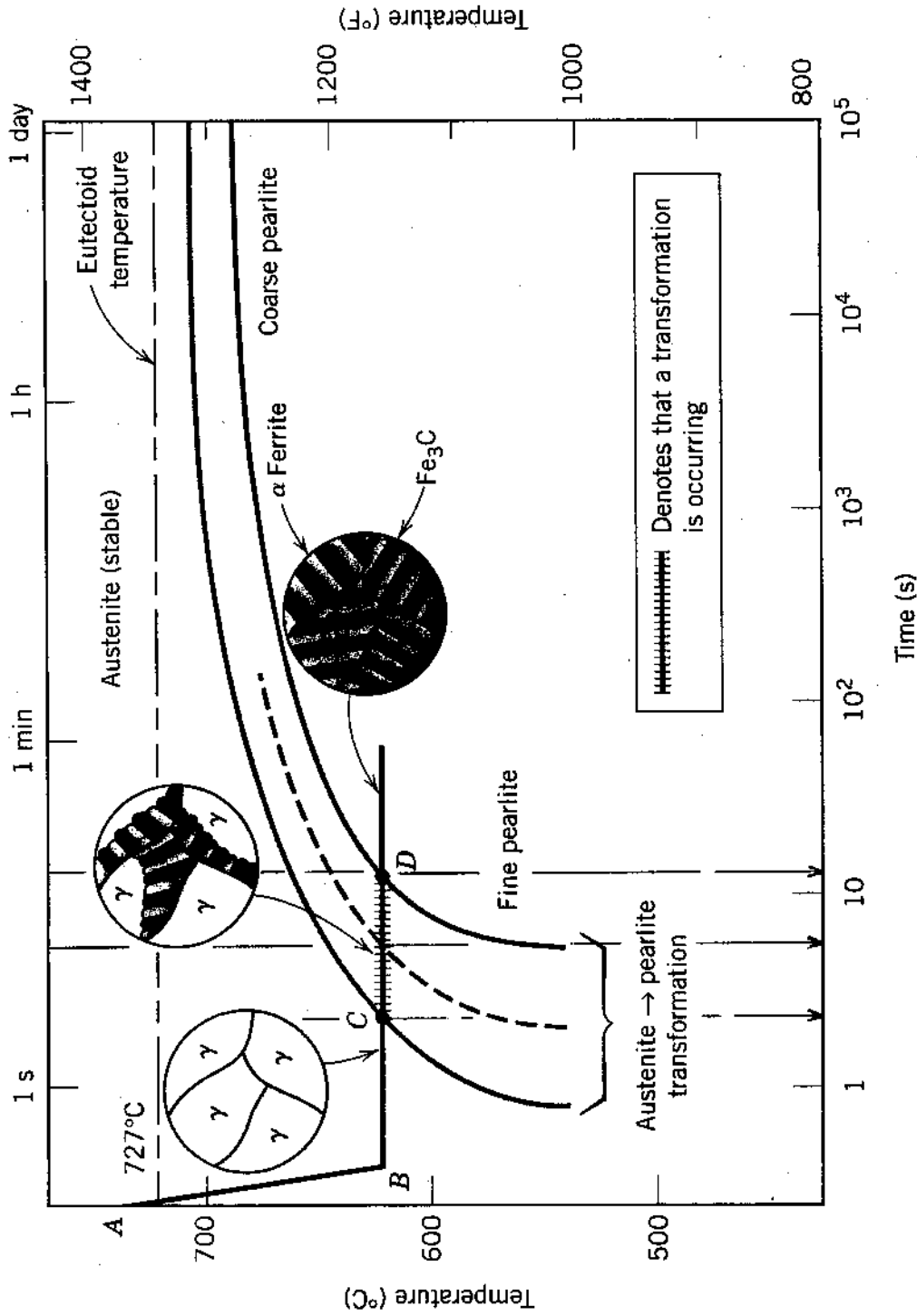


Figure 10.14 Isothermal transformation diagram for a eutectoid iron-carbon alloy, with superimposed isothermal heat treatment curve (ABCD). Microstructures before, during, and after the austenite-to-pearlite transformation are shown. [Adapted from H. Boyer (Editor), *Atlas of Isothermal Transformation and Cooling Transformation Diagrams*, American Society for Metals, 1977, p. 28.]

9. Phase-transformation kinetics can be suitably represented by the Avrami equation:

$$y = 1 - \exp(-kt^{1.7}) \quad (1)$$

Rearranging terms and taking logarithms,

$$-\ln(1-y) = kt^{1.7} \quad (2)$$

At  $t = 100$  sec,  $y = 0.5$ :

$$k = 0.6932 / (100)^{1.7} = 2.76 \times 10^{-4} \text{ sec}^{-1.7}$$

For  $y = 0.99$ ,  $t = t_{0.99}$ :

$$4.6052 = 2.76 \times 10^{-4} (t_{0.99})^{1.7}$$

$$t_{0.99} = 304.6 \text{ sec.}$$

10. The lamellar pearlite microstructure consisting of  $\alpha$ -Ferrite and  $\text{Fe}_3\text{C}$  undergoes a marked change when subjected to prolonged heat treatment (for 18-24 hrs) at about  $700^\circ\text{C}$ : the elongated  $\text{Fe}_3\text{C}$  phase becomes spherical-shaped  $\text{Fe}_3\text{C}$  distributed in the  $\alpha$ -ferrite matrix. The driving force for this change is decrease in the interfacial free energy: for a given volume, spherical-shape offers the lowest surface area. The much larger interfacial area of the lamellar pearlite (layers of  $\alpha$ -ferrite/ $\text{Fe}_3\text{C}$ ) gives away to the smaller area of  $\text{Fe}_3\text{C}$ /ferrite phase in the spheroidite. This is more ductile and less hard as compared to the fine pearlite.

11(a). Recrystallization-rate,  $r = \frac{1}{t_R}$  (1)

$\ln r = -\ln t_R = \frac{-Q}{R \cdot T} + \ln A$  (2)

At  $256^\circ\text{C}$  or  $529^\circ\text{K}$ :

$-\ln 100 = \ln A - \frac{Q}{529 R}$  (3)

At  $283^\circ\text{C}$  or  $556^\circ\text{K}$ :

$-\ln 10 = \ln A - \frac{Q}{556 R}$  (4)

Subtracting

$\ln \frac{10}{100} = \frac{Q}{R} \cdot \frac{529 - 556}{(556)(529)}$

$Q = (R \ln 0.1) 556(529) / (-27)$

(i)  $Q = 208,550 \text{ J/mole}$

From Eqn. (4), we find

$\ln A = -\ln 10 + \frac{208550}{8.3143 \times 556}$

(ii)  $A = 3.915 \times 10^{18} \text{ hr}^{-1}$

At  $T = T_3$ ,  $t_R = 2.5 \text{ hrs}$ .

$-\ln 2.5 = \ln (3.915 \times 10^{18}) - \frac{208550}{8.3143 T_3}$

(iii)  $T_3 = \frac{208550}{8.3143 \times 43.7276} = 573.6^\circ\text{K}$

11(b). Since age-hardening is a thermally-activated process,

$\ln \left( \frac{1}{t_{ag}} \right) = -\ln t_{ag} = \ln A_{ag} - \frac{Q_{ag}}{8.3143 \cdot T}$

Substituting

$-\ln 10 = \ln A_{ag} - \frac{Q_{ag}}{8.3143 \times 600}$

$-\ln 280 = \ln A_{ag} - \frac{Q_{ag}}{8.3143 \times 500}$

$Q_{ag} = \frac{8.3143 \cdot \ln 28}{(600 - 500)} (600 \times 500) = 83,115 \text{ J/mole}$



$$A_{ag} = 1.721 \times 10^6 \text{ hr}^{-1}$$

At  $T = 530^\circ\text{K}$ :

$$\ln \frac{1}{t_{ag}} = \ln 1.721 \times 10^6 - \frac{83115}{8.3143 \times 530}$$

$$- \ln t_{ag} = -4.5032 ; \underline{t_{ag} = 90.3 \text{ hrs}}$$

12. Yield strength of special steel = 1400 MPa

$$150 \text{ MPa} \cdot \text{m}^{1/2} = (1) 1400 (\pi a)^{1/2}$$

$$a = 0.003654 \text{ m} \approx 3.654 \text{ mm}$$

$$\text{length of inner crack} = 2a = \underline{7.308 \text{ mm}}$$

$$\text{surface crack-length, } a = \underline{3.654 \text{ mm}}$$

13. For MgO(S), fracture toughness = 3 MPa·m<sup>1/2</sup>

$$\text{surface crack-length, } a = 1 \text{ mm or } 10^{-3} \text{ m}$$

$$3 \text{ MPa} \cdot \text{m}^{1/2} = (1) \sigma_f (\pi \times 10^{-3} \text{ m})^{1/2}$$

$$\text{breaking strength, } \sigma_f = \underline{53.52 \text{ MPa}}$$

14. For the 1-mm rod drawn from plate glass,

$$\alpha_L, \text{ linear coeff. thermal exp.} = \frac{\Delta L}{L_0} / \Delta T$$

$$\Delta T = 180^\circ\text{K}$$

$$\text{strain, } \epsilon = \frac{\Delta L}{L_0} = 9 \times 10^{-6} \times 180 = 0.00162$$

$$\text{Elastic modulus, } E = \frac{\sigma}{\epsilon} = 70 \times 10^3 \text{ MPa}$$

$$\text{stress, } \sigma = E \epsilon = \underline{113.4 \text{ MPa}}$$

As the rod-length is a fixed  $L_0$ ,  $\sigma$  is tensile.

15. Consider  $1 \text{ cm}^3$  of composite:  $V_E = 0.5 \text{ cm}^3$ ;  $V_{\text{epoxy}} = 0.5 \text{ cm}^3$

$$\text{mass of E-glass} = 0.5 \times 2.54 = 1.27 \text{ g}$$

$$\text{mass of epoxy} = 0.5 \times 1.10 = 0.55 \text{ g}$$

$$\text{total} = 1.82 \text{ g}$$

$$\% \text{ E} = 1.27 \times 100 / 1.82 = 69.8\%$$

$$\% \text{ epoxy} = 0.55 \times 100 / 1.82 = 30.2\%$$

$$\rho_{\text{com}}, \text{ density of the composite} = \underline{1.82 \text{ g/cm}^3}$$

16.  $E_{\text{com}} = 0.33(5.9) + 0.67(360) = \underline{243.5 \text{ GPa}}$

$$E_{\text{com}}(\text{expt.}) = 221 \text{ GPa}; \text{ difference} = 10.2\%$$

Thus, the upper limit (Hashin-Shtrikman) modulus  $E_{\text{com}}$  shows good agreement with the experimental value.

17. For an intrinsic semiconductor,

$$\ln \sigma = \ln \sigma_0 - \frac{E_g}{2kT}$$

At  $T = 278^\circ \text{K}$ :

$$\ln 390 = \ln \sigma_0 - \frac{E_g}{2k(278)}$$

At  $T = 298^\circ \text{K}$ :

$$\ln 1010 = \ln \sigma_0 - \frac{E_g}{2k(298)}$$

Subtracting and solving,

$$\frac{E_g}{2k} = \ln\left(\frac{1010}{390}\right) \frac{298 \times 278}{(298 - 278)} = 3942; \underline{E_g = 0.679 \text{ eV}}$$

$$\ln \sigma_0 = \ln 390 + \frac{3942}{278}; \sigma_0 = 5.60 \times 10^8 \Omega^{-1} \cdot \text{m}^{-1}$$

At  $T = 288^\circ \text{K}$ :

$$\sigma_{288} = \sigma_0 e^{-13.686} = \underline{637.5 \Omega^{-1} \cdot \text{m}^{-1}}$$

$$18. \text{ molar volume, } V_m = \frac{28.09 \times 10^{-3} \text{ kg/mole}}{2330 \text{ kg/m}^3}$$

$$V_m = 1.2056 \times 10^{-5} \text{ m}^3/\text{mole}$$

$$(a) \quad n_{Si} = \frac{6.022 \times 10^{23} \text{ si/mole}}{1.2056 \times 10^{-5} \text{ m}^3/\text{mole}} = \frac{N_A}{V_m}$$

$$n_{Si} = 4.995 \times 10^{28} \text{ si/m}^3$$

(b) The conductivity ( $\sigma$ ) is given by

$$\sigma = n \mu_n q$$

$$n = \text{electron concentration, } e^-/\text{m}^3$$

$$\mu_n = \text{electron mobility} = 0.19 \text{ m}^2/\text{V}\cdot\text{s}$$

$$q = \text{charge on } e^- = 1.6022 \times 10^{-19} \text{ coulomb}$$

Therefore

$$100 \Omega^{-1} \text{ m}^{-1} = n (0.19) 1.6022 \times 10^{-19} \frac{\text{m}^2 \cdot \text{coulomb}}{\text{V}\cdot\text{s}}$$

$$n = 3.285 \times 10^{21} e^-/\text{m}^3$$

Units:

$$\frac{\Omega^{-1} \text{ m}^{-1} \text{ V}\cdot\text{s}}{\text{m}^2 \cdot \text{coulomb}} = \frac{\text{Amp}\cdot\text{s}\cdot\text{m}^{-1}}{\text{m}^2 \cdot \text{coulomb}} = \text{m}^{-3}$$

19. The temperature-drop induces respective forces  $F_B$  and  $F_p$  in the borosilicate ( $\alpha_B = 2.7 \times 10^{-6} \text{ K}^{-1}$ ) and plate glass ( $\alpha_p = 9 \times 10^{-6} \text{ K}^{-1}$ ).

$$F_B + F_p = 0$$

The structure sustains a strain given by

$$\epsilon = \epsilon_B = \left( \frac{\Delta L}{L} \right)_B = \epsilon_p = \left( \frac{\Delta L}{L} \right)_p$$

In general,

$$\frac{\Delta L}{L} = \alpha \cdot \Delta T + \frac{\sigma}{E}$$

For borosilicate,

$$\text{area of cross-section, } A_B = \frac{\pi}{4} (10^{-3} \text{ m})^2$$

$$A_B = 0.7854 \times 10^{-6} \text{ m}^2$$

$$\sigma_B = \frac{F_B}{A_B} = 1.2732 \times 10^{10} F_B$$

$$\left(\frac{\Delta L}{L}\right)_B = -180 \times 2.7 \times 10^{-6} + \frac{1.2732 \times 10^{10}}{70 \times 10^9} F_B \quad (\text{I})$$

For plate glass,

$$\text{area, } A_P = \frac{\pi}{4} [(1.2 \times 10^{-3})^2 - (10^{-3})^2]$$

$$A_P = 0.3456 \times 10^{-6} \text{ m}^2$$

$$\sigma_P = \frac{F_P}{A_P} = \frac{-F_B}{A_P} = -2.8935 \times 10^6 F_B$$

$$\left(\frac{\Delta L}{L}\right)_P = -180 \times 9 \times 10^{-6} - \frac{2.8935 \times 10^6}{70 \times 10^9} F_B \quad (\text{II})$$

Setting Eqn. (I) equal to Eqn. (II),

$$\frac{4.1667 \times 10^6}{70 \times 10^9} F_B = -180 \times 6.3 \times 10^{-6}$$

$$F_B = -19.05 \text{ N}$$

and

$$F_P = +19.05 \text{ N}$$

Stresses:

$$\sigma_B = F_B / A_B = -24.26 \text{ MPa (Compr.)}$$

$$\sigma_P = F_P / A_P = +55.12 \text{ MPa (tension)}$$

Surface-layer in tension; this would lead to cracking of the structure.